RPN80

FLUTE INSTABILITY OF A PARAPOTENTIAL SHEET BEAM

by

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This note is intended to indicate an instability mode which could adversely affect the behavior of high-current parapotential beams and which may, according to evidence which will be presented, have produced the marked tendency of the Gamble-I spot to move radially away from the center as well as similar behavior in other high-current generators. The theory presented will be of an extremely crude, order-of-magnitude character. A suggested instability cure, together with evidence of its effectiveness, will also be indicated.

First we assume that the space-charge-limited flow confined between a surface of maximum potential[1] and the cathode is replaced for simplicity by a single sheet. In a conical geometry this sheet is a cone at a position intermediate between the cathode and the anode (figure 1a). Second we unwrap and stretch the cone-shaped beam into a plane as in figure 1b. We consider only perturbations sinusoidal in y and of infinite wavelength in z.

We examine first the force on a single beam electron due only to the external electrodes (subscript 1). We have to first order

$$(\gamma m\ddot{x})_1 = e(E_1 + \beta H_1),$$

where H_1 results from current in the cathode sheet. Further,

$$\beta = (1 - \gamma^{-2})^{\frac{1}{2}},\ \gamma = \gamma_o + eE_1 x/mc^2.$$

So

$$\beta = \left[1 - (\gamma_o + eE_1 x/mc^2)^{-2}\right]^{\frac{1}{2}}$$

$$\sim (1 - \gamma_o^{-2} + 2\gamma_o^{-3} eE_1 x/mc^2)^{\frac{1}{2}}$$

$$\sim \beta_o(1 + eE_1 x/\beta_o^2\gamma_o^3).$$

Thus

$$\ddot{x}_1 = e\left[E_1 + \beta_o H_1(1 + eE_1 x/\beta_o{}^2\gamma_o{}^3 mc^2)\right]/\gamma_o m$$

$$= (e^2E_1H_1/\beta_o\gamma_o{}^4m^2c^2)x$$

on putting $E_1 + \beta_o H_1 = 0$.

Now suppose the sheet is deformed sinusoidally in the y-direction by a perturbation having the radian length $\lambdabar$. The force due to this perturbation on an electron at the crest (displacement x) is of the form

$$(\gamma m\ddot{x})_2 = K\sigma e^2 x/\lambdabar\gamma^2$$

for $x \ll \lambdabar$; here K is a constant (order of unity) representing the proper calculation of the integral involved, σ is the electron density per unit surface of the beam, and the γ^2 in the denominator allows for the magnetic attraction. The form of this expression is clear from consideration of a line charge $\lambdabar\sigma e$ on the equilibrium plane, producing an electric field $2\lambdabar\sigma e/\lambdabar = 2\sigma e$ at a distance $\lambdabar$, of which the x-component is $2\sigma e(x/\lambdabar)$. Adding the forces with "1" and "2" subscripts, we have for the harmonic oscillator radian frequency squared

$$\omega^2 = -e^2E_1H_1/\beta\gamma^4m^2c^2 - K\sigma e^2/\gamma^3 m$$

$$\sim (c/\gamma)^2\left[-E_1H_1/1700^2\beta\gamma^2 - K(\nu/\gamma)(1/2\pi R)(2\pi M/R)\right]$$

$$= (c/\gamma)^2\left[(E_1/1700\beta\gamma)^2 - KM\nu/\gamma R^2\right],$$

where we have put $\sigma e^2/mc^2 \equiv \nu/2\pi R$ (thus identifying ν with the total beam linear density), given the azimuthal mode number the symbol M, and dropped the zero subscripts in first order. Instability occurs when the bracket is negative, with a growth rate

$$1/\tau = (c/\gamma)\left[KM\nu/\gamma R^2 - (E_1/1700\beta\gamma)^2\right]^{\frac{1}{2}}.$$

Some numbers: putting R=1 cm, K=M=1, $\beta\gamma=3$, $E_1=3000$ esu, $\nu/\gamma=10$ gives

$$1/\tau = (c/3)(10/3 - 1/3)^{\frac{1}{2}}.$$

Thus growth times, if the first term greatly exceeds the second, are of the order of light propagation times. There is no hope of stopping the instability by its going at an inherently slow rate. It is necessary instead to keep the focusing term dominant over the defocusing one at all interesting parameter values.

The following general conclusions can be drawn from the analysis, such as it is:

1. It is desirable to keep spacings close, which enhances E_1 and thus the focusing force. In a strictly conical geometry, where $E_1 \propto 1/R$, a given mode is stable everywhere if it is stable anywhere. In a real situation, however, this condition must always be departed from at small R.

2. If the motion is definitely unstable, its growth rate is about

$$1/\tau \sim (c/\gamma R)(KM\nu/\gamma)^{\frac{1}{2}}.$$

The time τ is entirely negligible compared to the pulse length.

3. Only M=1 and M=2 have been observed, although theoretically higher modes have faster growth rates. The formulation

$$\text{force}_2 = K(e/\gamma)^2(\sigma x/\lambda\!\!\!^-)$$

fails for amplitudes comparable to $\lambda\!\!\!^-$, since the linear analysis no longer applies. If we take a maximum amplitude of 1 cm, this limit corresponds to a wavelength of 6.28 cm (or a radius of 1 cm for M=1, 2 cm for M=2, etc., since $\lambda\!\!\!^- = R/M$). Thus the higher-order modes are always very unstable but may be limited in amplitude to about R/M. This may explain the dominance of low-order modes.

4. An obvious cure for the instability, if one believes this, is to increase the minimum value of R so as to make the first term of $(1/\tau)^2$ less in magnitude than the second for all $M < R/\hat{x}$, where $\hat{x}$ is the permissible instability amplitude. Thus

$$R > (1700\beta\gamma/E_1)^2 K\nu/\gamma\hat{x}.$$

If $\hat{x} \sim R$, this reduces to

$$R > (1700\beta\gamma/E_1)(K\nu/\gamma)^{\frac{1}{2}}.$$

This is in some sense the requirement that the discharge should remain in a circle of radius R. We should expect, however, that there could be considerable structure within the circle over which we have no control.

Experimental evidence for the unstable flow and its suppression is shown in figures 2, 3, and 4. Figure 2a shows typical anode damage with a conical cathode provided with a needle. The cathode is shown in figure 2b. Tantalum anodes in these cases generally show a destroyed region which increases in width toward the edge. Figure 3a shows results with a central projection but with a ring affixed to the surface just outboard of the projection. The damage indicates a lessening of the tendency of the discharge to move radially, and there is marked improvement in shot-to-shot reliability. The cathode in this case is shown in figure 3b. To effect the greatest possible increase in radius the entire central portion of the cathode was removed, and the results are shown in figure 4. The first group represent an 8-mm ring-to-anode spacing; the second corresponds to a 5-mm spacing. The enhancement of damage and confinement at the smaller spacing (larger E_1) is evident. Some shots at the larger spacing reveal the M=2 mode. This as well as residual M=1 behavior, can usually be traced to inadequate cathode cleaning. A series of shots with the central portion of the cathode made flush with the base of the ring outside showed this configuration to be less effective than that in which the cathode is entirely hollow.

It is clearly necessary to obtain more evidence on this effect, but what is already at hand shows that the general curative procedures suggested by the foregoing tentative theory are correct. The theory can obviously be greatly improved.

REFERENCES

1. David C. dePackh, "Parapotential Flow", Radiation Project Progress Report Number 5, 24 June 1968.

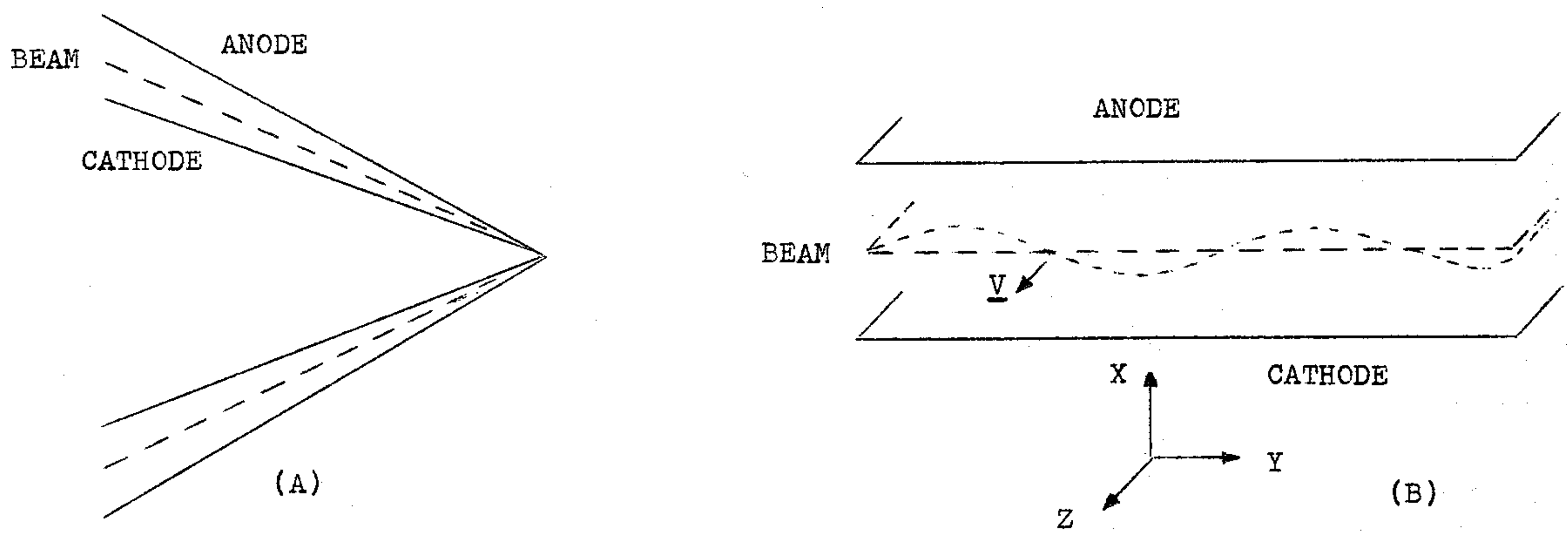
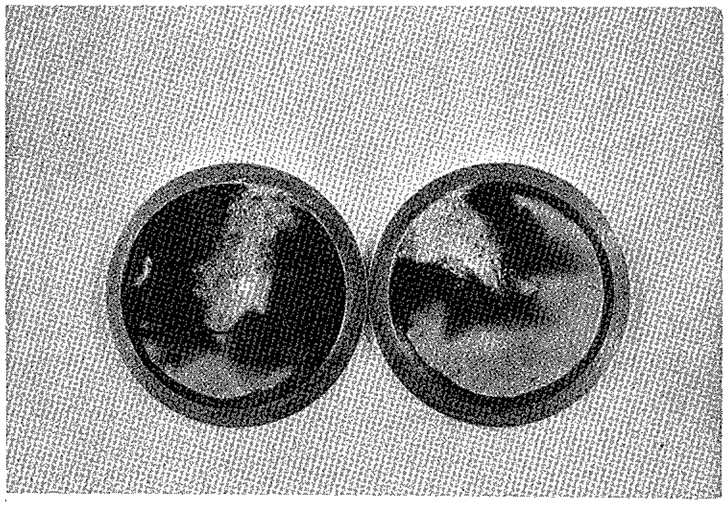
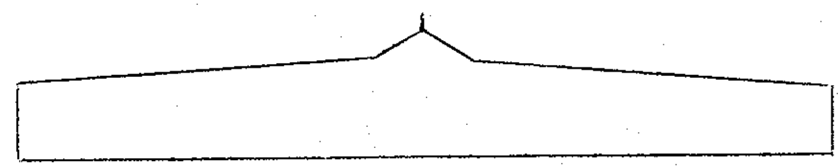


Figure 1. (A) Conical geometry showing sheet parapotential beam. (B) Plane geometry simulating conical flow and showing equilibrium and perturbed beams



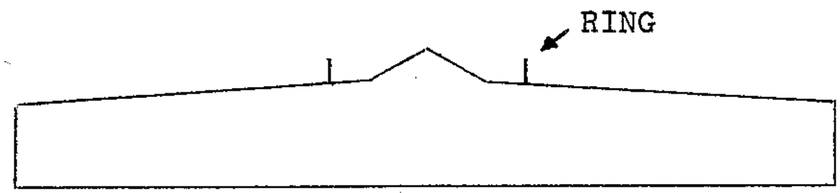
(A)



(B)

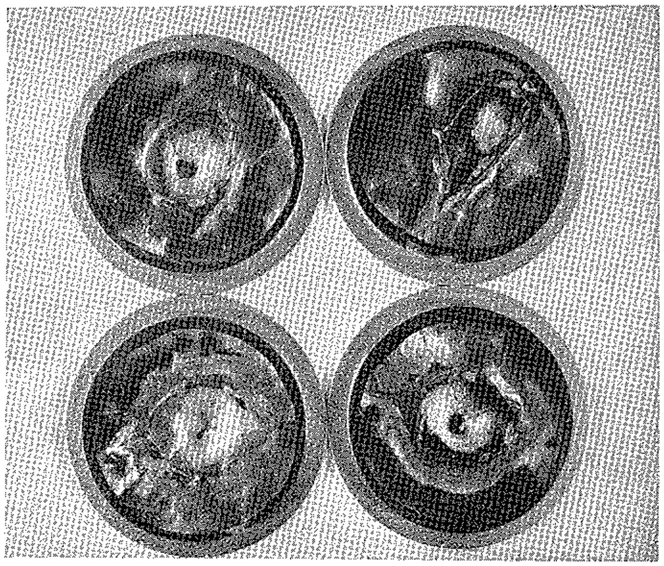
Figure 2. (A) Anode damage with cathode configuration 2b showing severe radial instability.
(B) Cathode producing results of 2a. Diameter is 4.25 inches

(A)

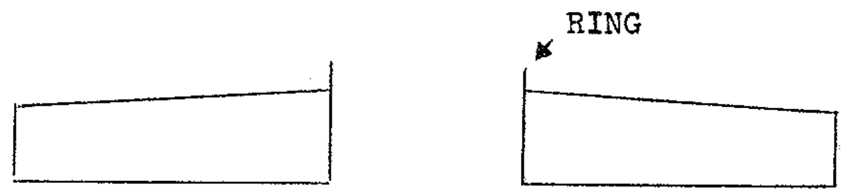


(B)

Figure 3. (A) Anode damage with cathode configuration 3b. Disc diameter is 4 inches.
(B) Cathode producing results of 3a.



(A)



(B)

Figure 4. (A) Top two: anode damage for cathode 4b at 8 mm cathode-anode spacing. Lower two: Same at 5 mm cathode-anode spacing. Tantalum targets are still attached. Note M = 2 effect in upper right-hand picture. (B) Cathode producing results of 4a.